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NONLINEAR CONTINUUM MECHANICS & APPLIED ANALYSIS

Program Director: A. Nachman. Principal Investigator: J.H. Maddocks.

FINAL REPORT JUNE 1989

§1 Papers Appearing or Submitted in 1986-89

"Ropes in Equilibrium" (with J.B. Keller), SIAM J. Appl. Math. **47** (1988) pp. 1185-1200. This paper is concerned with the equilibria of ropes or strings that are lying on surfaces or intertwined with other strings. The crucial feature is the presence of friction. Some analysis is made of knots and hitches, and in particular criteria are given that determine when some knots will hold. Certain optimization problems are solved to determine all equilibria consistent with various classes of loading. This line of investigation is now being extended to sliding configurations with applications in robotics.

"Stability and folds", Arch. Rat. Mech. Anal. **99** (1987) pp. 301-328. This work strengthens standard stability exchange results for one parameter bifurcation problems when it is further assumed that the problem at hand possesses a variational structure. Moreover it is shown that when the problem has a constrained variational structure, the appropriate notion of stability can still be determined from the shape of the solution branch in a certain bifurcation diagram. This last result involves an application of my abstract results obtained in "Restricted quadratic forms and their application to bifurcation and stability in constrained variational principles", SIAM J. Math. Anal. **16** (1985) pp. 47-68. Extensions of these results to multiple parameter problems and to proving stability of relative equilibria in Hamiltonian dynamical systems is a mainstay of my current research (see below).

"A Model for Disclinations In Nematic Liquid Crystals", in *"Theory and Applications of Liquid Crystals"*, IMA Volumes in Mathematics and Its Applications, **5**, Springer-Verlag (1987) pp. 255-269. The analysis of point defects in liquid crystals is currently the subject of much mathematical research. However many of the defects actually seen in experiments have the appearance of line singularities or *disclinations*. Such disclinations can be constructed as singular solutions of the equilibrium equations according to the Frank-Oseen model, but they are often associated with infinite energy. This paper offers one resolution of this apparent paradox by the construction of finite-energy disclination solutions to a strain gradient modification of the Frank-Oseen equations that was proposed by Ericksen. The analysis forces through the method of upper and lower solutions for monotone operators to obtain the existence and qualitative features of equilibria modelling disclinations that exhibit phase transitions. The main point of mathematical interest is that the existence proof includes a geometrical construction that measures, in a certain sense, the size of the potential barrier between two phases. According to this analysis, in order that two phases can be linked it is necessary that the barrier, or spinoidal region, is not too large. It is this feature that may extend to other applications. To my knowledge, this analysis is one of the first results in strain-gradient theories that quantifies the importance of the intuitive notion of the size of the potential barrier.

"On the Maneuvering of Vehicles" (with J.C. Alexander) SIAM J. Appl. Math. **48** (1988) pp. 38-52. The issue addressed here is the kinematics governing the motion of vehicles that move on rolling wheels. Surprisingly this comparatively simple problem has been the subject of erroneous analysis by several authors. The main point of study is the connection between the

steering controls and the position and orientation of the vehicle in space. In mathematical terms the work comprises a new Euler-Savary formula, that is, a formula connecting the centre of rotation of a rigid body with the centres of curvature of the paths traced by points of the body. Further applications of this work in the field of robotics are described below.

"Restricted Quadratic Forms, Inertia Theorems, and the Schur Complement", Linear Algebra and Appl. 108 (1988) pp. 1-36. This presentation comprises a finite dimensional version of the Hilbert space theory described in my paper "Restricted Quadratic Forms and Their Application ...". Naturally, specialization to finite dimensions allows a considerable simplification and extension of the general Hilbert space theory. Interestingly, this new finite-dimensional work contains as special cases many disparate results in the areas of inertia theorems for matrices, and of duality results in nonlinear programming. The new work reveals the necessity of several, apparently unnatural, hypotheses that were made in previously known theorems, and connections between apparently unrelated results are exposed.

"On the Kinematics of Wheeled Mobile Robots" (with J.C. Alexander) accepted by, and in press at, the International Journal on Robotics Research. The first part of this work is a continuation of our analysis of the steering of vehicles under the hypothesis of ideal rolling, but with the focus shifted to the particular context of wheeled robots. The first issue is to obtain the position and orientation of the robot body given the time history of the steering and rotation rates of various wheels. Control strategies for obtaining desired paths can also be found, and different designs of robots that have actually been constructed can be compared for controllability. The second line of investigation arises when the steering and speeds of the controlled wheels are not compatible with ideal rolling. The ensuing motion can then only be determined by undertaking an analysis of sliding friction. We show that a quasi-static formulation is equivalent to a minimization principle that is analogous to Rayleigh's Dissipation functional. In the context of sliding friction one is led inexorably to the minimization of a convex, nonsmooth, nondifferentiable functional. The further understanding of this functional is an active line of research. In particular essentially the same functional arises in sliding problems, such as a rope being dragged over a rough plane, or a robot manipulating objects by pushing, that are completely separate from rolling problems.

"On second-order conditions in constrained variational principles", submitted to Proc. R. Soc. (London) A. In this work I combine results from "Restricted Quadratic Forms and Their Application ..." with a multi-parameter version of the results appearing in "Stability and folds" to obtain a comprehensive approach to the analysis of the second-order tests that determine which extremals of multiply constrained isoperimetric variational principles are actually constrained minima. The results are couched in terms of the shape, more specifically the singularities and curvatures, of the bifurcation hyper-surface that is obtained when a modified Lagrangian (evaluated on solutions) is plotted above the Lagrange multipliers associated with the constraints. The results are applied to determine the stability of certain satellite motions. Further applications and ramifications are described below.

§2 Progress in Ongoing Projects

I here describe projects in which partial, but as yet unpublished, results have been obtained. It is intended that work on these projects will be continued under AFOSR Grant 89-0376

"Optimal Design of Columns Against Buckling" In a seminal work Tadjbaksh & Keller gave a simple argument that determines the shape of the column of given volume that maximizes

the first buckling load. However, their analysis contains the implicit assumption that the eigenvalue corresponding to the optimal buckling load is simple. It was subsequently pointed out by Olhoff & Rasmussen and others that for some boundary conditions this assumption is unwarranted: the optimal eigenvalue can be double. Olhoff & Rasmussen describe a numerical procedure to treat this, so called, bimodal case. I have shown that the method of Keller & Tadjbaksh can be adapted to treat the bimodal case analytically, and a closed form expression for the optimal shape is obtained.

Although the above described analysis has been completed it has not been submitted for publication. The reason is that it has become apparent that the actual model, which is also adopted in a large number of other papers, has a potential flaw. The difficulty is that the optimization procedure leads to column shapes whose thickness vanishes at various points. While the physical relevance of such solutions has previously been questioned, it does not appear to have been pointed out that the vanishing cross-section can give rise to an "eigenvalue" problem that may have continuous spectrum. In mathematical terms a differential equation with coefficients that are not L^1 is obtained. Such an equation need not have purely point spectrum, so the meaning of several prior works, which maximize the lowest eigenvalue, is called into question. It appears that the difficulties can be finessed by a singular point analysis, combined with the avoidance of various "standard" transformations, but the resolution has yet to be completed.

"Director theories of rods" (joint with S.S.Antman) This analysis reduces the equilibrium conditions for a uniform isotropic rod to a phase-plane for the curvature of the centerline of the rod. The phase-plane has three distinct qualitative forms that depend on four constants that reflect the possible loadings, twist etc. While the problem has previously been reduced to a single phase-plane involving an Euler angle, the present treatment avoids artificial difficulties associated with the polar singularity. The particular problem in mind is a model for the structure of super-coiled DNA. The model comprises an elastic rod that is closed on itself in a loop after a specified amount of twist is imposed. It is therefore important to address the question in differential geometry of obtaining criteria in terms of curvature and torsion for a line to be closed. While this problem is intractable in general, the equilibrium conditions associated with the rod imply restrictions that appear to allow headway.

"The Stability of Relative Equilibria"-Many important Hamiltonian systems have periodic solutions that are associated with symmetries of the equations. The equations governing the rotation of heavy rigid bodies comprise one such system in which the special solutions are known as permanent rotations. While it is well known that stationary solutions of a Hamiltonian system can be characterized as extremals of the potential energy, it is less widely appreciated that symmetry-related periodic solutions, or relative equilibria, can also be given a variational characterization. This variational characterization is important because if a periodic solution is associated with a minimizer (in some sense), as opposed to merely being a stationary point, then a stability result is very often available.

We are therefore left with the problem of characterizing those extremals of a variational principle that are actually local minima. This is the province of second-order necessary and sufficient conditions. The special feature of the variational principles that arise here is that they invariably involve isoperimetric side constraints on the conserved quantities of the dynamical system. For example, permanent rotations of a rigid body can be characterized as extremals of the energy subject to prescribed values of two components of the angular momentum. Because isoperimetric constraints are involved, the second-order conditions for a local minimum will typically *not* reduce to the simple case of definiteness of a quadratic form. There are several (more or less) related methods to approach the problem. One technique is described in my recent paper "On second-order conditions in constrained variational principles". The tests described there seem particularly suited

to determining stability of equilibrium solutions to noncanonical Hamiltonian dynamical systems of the form

$$\dot{u} = J(u)\nabla H(u).$$

Here $J(u)$ is a *singular* skew-symmetric operator, and $H(u)$ is the Hamiltonian depending on the variable u . Perhaps surprisingly, many important physical systems can be cast in this abstract form. Moreover the nullspace of J , enoyted $N(J)$ is typically spanned by vectors ∇C_i that are gradients of scalars $C_i(u)$. The functionals $C_i(u)$ are the *Casimirs*. Necessarily they are conserved quantities of the dynamical system. Equilibria satisfy the equation $\dot{u} = 0$ which is equivalent to $\nabla H(u) \in N(J)$ or

$$\nabla H(u) = \sum_i \lambda_i \nabla C_i(u),$$

which equation immediately leads to the variational principle of minimizing H subject to each of the C_i held fixed and the constants λ_i can be interpreted as Lagrange multipliers. The ramifications and application of this formulation are an active line of research.

§3 Reports on Visits and Conferences

Two major visits were supported by this contract.

(i) Special year on Phase Transitions and the Calculus of Variations, Heriot-Watt University. I visited Heriot-Watt for an extended period this summer and delivered a lecture on stability predictions in variational bifurcation problems. The visit was valuable for the interaction with the other participants, who comprised the majority of the Western experts on phase transitions. I was involved in extensive discussions concerning the implications of one-dimensional static models for both dynamic and multi-dimensional problems.

(ii) International Symposium On Nonlinear Seismology, Suzdal, near Moscow. I attended this conference as the guest of the Soviet Academy of Sciences, and delivered a survey and introductory lecture to mathematical models of phase transitions. While the bulk of the participants were seismologists, there was also a group of applied mathematicians working on problems of phase transitions. As well as obtaining a valuable introduction to the Soviet literature on phase transitions, I also learned of new problems. For example, there is considerable interest in modelling the internal structure of planets under self-gravitation by assuming that pressure has a nonmonotone dependence on density. The indisputable assumption of spherical symmetry then leads to a novel one-dimensional problem of phase transitions of a type that has not been discussed in the mathematical literature.

